Composite clock : New simulation's results obtained from an algorithm developed for locking a VCO to HM clock and then to Cs frequency standard.

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SUMMARY *We present the algorithm developed for locking a VCO to HM clock and to Cs clock in order to provide a very stable system having: the mid-term performances of the HM clock, the long term stability of the Cs clock and also the VCO's caracteristics at the very short term. So it's called composite clock.*

INTRODUCTION

This algorithm takes a very important role in this composite clock. Indeed, it permits us to control the VCO according to the HM clock and to the Cs clock. The architecture of this system is described in figure 1. And we remark 2 important steps:

- The clock intercomparison measures gives the comparisons between frequencies standard by using the DMTD's method.

- The controlling algorithm uses these comparisons for locking the VCO to the HM clock and then to the Cs beam frequency standard.

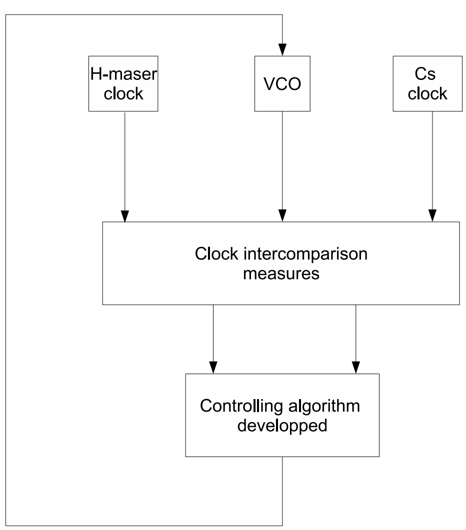

Figure 1: Architecture of our composite clock.

In this article we are treated only the controlling algorithm. We will talk about his principle and his theorical study. Simulations results will be also shown.

1. The controlling algorithm developed

1.1 The servoing loop

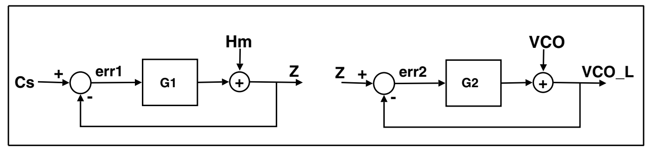

Figure 2: Controlling loop.

Figure 2 shows the principle of the servoing loop. Note that it'snt an electronic device built, but just a virtual concept which permits us to determinate the command to apply to the VCO. We use 2 loops:

1. The first loop generates a virtual signal Z corresponding to a virtual locking of the HM clock to the Cs clock. It corresponds to the slowest loop. And we have:

$$\mathrm{Z} = \frac{\mathrm{HM}}{1+\mathrm{G_1}} + \frac{\mathrm{G_1} \times \mathrm{Cs}}{1+\mathrm{G_1}}$$

2. The second loop permits to control the VCO according to this virtual signal Z. It's produced a signal VCO_L corresponding to the locked VCO. This loop is the fatest loop. We obtain:

$$\mathrm{VCO_L} = \frac{\mathrm{VCO}}{1+\mathrm{G_2}} + \frac{\mathrm{G_2.Z}}{1+\mathrm{G_2}}$$

Combining the 2 equations we obtain:

$$\mathrm{VCO_L(z)} = \frac{\mathrm{VCO(z)}}{1+\mathrm{G_2(z)}} + \frac{\mathrm{G_2(z)\cdot HM(z)}}{[1+\mathrm{G_2(z)}][1+\mathrm{G_1(z)}]} + \frac{\mathrm{G_2(z)\cdot G_1(z)\cdot Cs(z)}}{[1+\mathrm{G_2(z)}][1+\mathrm{G_1(z)}]}$$

This relationship defines a Linear Time-Invariant (LTI) system with three inputs VCO, HM and Cs where :

Cs is the Cs clock signal.

HM represents the HM clock signal.

G_1 the transfert function of the first loop.

Z is the output signal of the first loop.

G_2 is the transfert function of the second loop.

VCO corresponds to the signal of the free running VCO.

VCO_L the signal of the locked VCO signal.

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1.2 Conditions about transfert functions G_1 and G_2

Loops have to allow to the locked signal to follow the references, so G_1 and G_2 must satisfy the 2 following conditions in the short term ($p \to \infty$).

$$\lim_{p\to\infty} \frac{1}{1+G_1(p)} = \lim_{p\to\infty} \frac{1}{1+G_2(p)} = 0$$

$$\lim_{p\to\infty} \frac{G_1(p)}{1+G_1(p)} = \lim_{p\to\infty} \frac{G_2(p)}{1+G_2(p)} = 1$$

And the 2 other conditions in the long term ($p \to 0$):

$$\lim_{p\to 0} \frac{1}{1+G_1(p)} = \lim_{p\to 0} \frac{1}{1+G_2(p)} = 0$$

$$\lim_{p\to 0} \frac{G_1(p)}{1+G_1(p)} = \lim_{p\to 0} \frac{G_2(p)}{1+G_2(p)} = 1$$

$p(= j\omega)$ represents the Laplace's variable.

G_1 and G_2 have the same role and the same mathematical form. Both are a low pass second order IIR digital filters but they have'nt the same constante time. G_1 represents the transfert function of the first loop or lowest loop and G_2 is the transfert function of the second loop or fatest loop.

We work in Z-domain, so G_1 and G_2 depend on the discret time parameter z. And we choose

$$G_{1/2}(z) = \frac{b_0 + b_1 z^{-1}}{(1-z^{-1})(1-z^{-1})} = \frac{b_0 + b_1 z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

For the stability of the system, b_0 and b_1 must satisfy the conditions: $\mathbf{-2 < b_1 < 0}$ and $\mathbf{-b_1 < b_0 < 4 + b_1}$. Figure 3 shows the stability domain of our system according to b_0 and b_1.

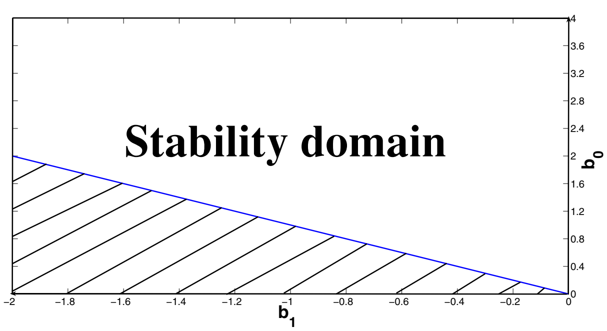

Figure 3 : Stability domain.

1.3 The controlling algorithm developed

We have 2 solutions for developing our algorithm:

1. Using the expression of the locked VCO given by:

$$\mathbf{VCO_L(z)} = \frac{\mathbf{VCO(z)}}{\mathbf{1+G_2(z)}} + \frac{\mathbf{G_2(z)\cdot HM(z)}}{\mathbf{[1+G_2(z)][1+G_1(z)]}} + \frac{\mathbf{G_2(z)\cdot G_1(z)\cdot Cs(z)}}{\mathbf{[1+G_2(z)][1+G_1(z)]}}$$

We could using the properties of the Z-transform write the expression of $VCO_L(z)$ in discrete time domain. But, we need to express $G_1(z)$ and $G_2(z)$ in the same time base.

2. The second solution is more easier than the first. Firstly, we determinate the expression of the signal error err_2 according to the intercomparisons between oscillators.
Calculus gives :

$$err_2(z) = HM(z) - VCO_L(z) + \frac{G_1(z)}{1+G_1(z)}[Cs(z) - HM(z)]$$

We remark that $\mathbf{err_2}$ depends on the intercomparison $(HM(z) - VCO_L(z))$ between the HM clock and the locked VCO.
And also on the term $\frac{G_1(z)}{1+G_1(z)}[Cs(z) - HM(z)]$ where $(Cs(z) - HM(z))$ is the comparison between the HM clock and the Cs clock.

Secondly with the properties of Z-transform, we calculate in discrete time domain the command to apply on the VCO according to the complete flow chart described in figure 4.

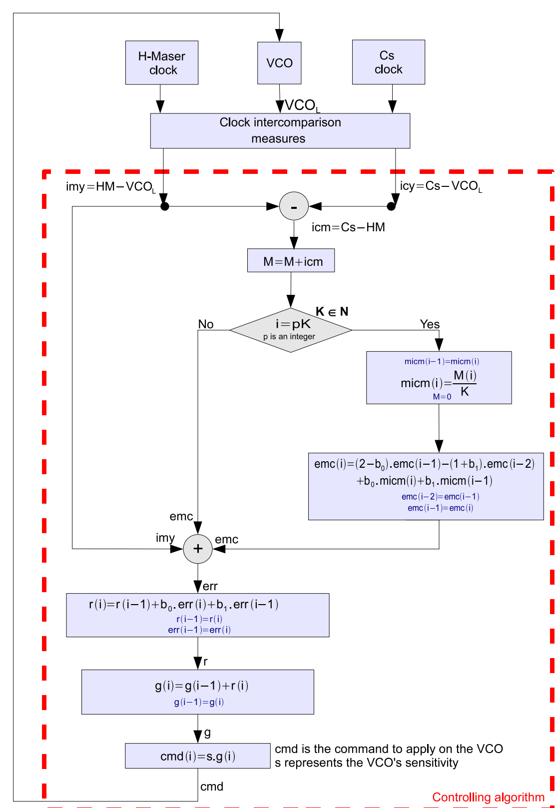

Figure 4 : Complete flow chart of the algorithm.

Figure 4 shows that we need 2 values (imy and emc) for having the signal error err to correct and to apply on the VCO.

1- imy could be derminate easily. It's just the intercomparison between the HM clock and the VCO given by the clock intercomparison measures.

2- emc is calculated from icm the comparison between the Cs standard and the HM clock according to :

- if $\mathbf{i} = \mathbf{pK}$, we average icm over the time K before to correct it by the function $\frac{\mathbf{G_1}}{\mathbf{1+G_1}}$ in order to find emc.
- or else we take the previous emc.

3- We add imc and emc for having the total signal error err which is corrected by the function $\frac{\mathbf{b_0+b_1 \cdot z^{-1}}}{\mathbf{1-z^{-1}}}$ and after by an another one $\frac{\mathbf{1}}{\mathbf{1-z^{-1}}}$. The result multiplied by the sensitivity (s) of the VCO gives the correction (cmd) to apply to the VCO.
We have defined 3 important parameters for our algorithm: K, (b_0, b_1) and s, so it could be interrested to study their effect on the stability of our system.

2. Simulation results and discuss

2.1 Simulation part 1

In this simulation part 1, we generate $N = 10^6$ data of normalised frequency deviations for each frequency standard.
We simulate the controlling algorithm using the process described in figure 3 and we trace in figure 5 the Allan deviation curves corresponding.

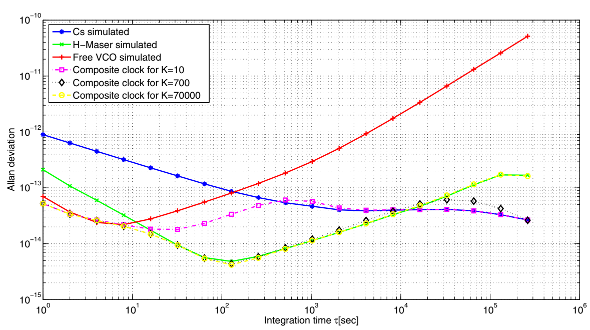

Figure 5 : Allan deviation curves of simulated oscillators and locked VCO for 3 values of K.

Note that data used in simulation part 1 are not very realistic. Indeed the simulated stability quartz is far better than any available quartz and between the intersections of the simulated HM clock with the simulated VCO and the simulated Cs clock with the simulated HM clock, we have 3 decades instead of 6 decades.

These conditions make simulation easier for us because we have fewer data, so simulation doesn't take much time. Then we can make others simulations in order to see the effect of the 3 parameters (K, (b_0, b_1) and s). While the algorithm is tested in conditions worse than real. We use a sample of 1s because we need the crossing between the simulated HM clock and the simulated VCO for showing well the effect of the fatest loop.

In figure 5, it can be seen the evolution of the stability for our three simulated oscillators (VCO, HM clock and Cs clock) and also for three values of K the ADEV curves of our locked VCO and we remark:

- if K is small $(K = 10)$, the VCO cannot be locked on simulated HM clock.

- if K is great $(K = 7000)$, the VCO is locked on HM clock beyond our need.

We conclude that K acts on the time constant of the slowest (first) loop. The best compromise in this simulation is given by $K = 700$.

2.2 Simulation part 2

1. In this new simulation part, we have $N = 10^{10}$ data of frequency deviations for each frequency standard. So, simulations take much time but they are more realistic than the previous shown in 5.

2. We applie always the process described in figure 4 for having the command to apply on the VCO.

3. And we trace the Allan deviation curves for each atomic clock and also for the locked VCO.

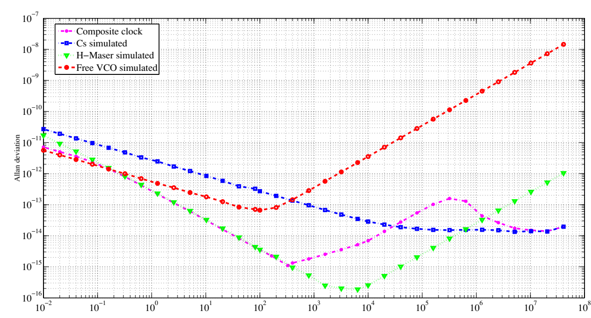

Figure 6 : New simulation's results.

Figure 6 shows the Allan deviation curve of our composite clock (e.g VCO's output) for $(\mathrm{b_0} = 4.4 \cdot 10^{-2}, \mathrm{b_1} = -4.25 \cdot 10^{-2})$, $\mathrm{K} = 10^5$ and $\mathrm{s} = 6$.
It can be seen that, with the algorithm described in figure 3, the locked VCO follows at first the free VCO, then (at $\mathrm{t} = 1 \cdot 10^{-1}\mathrm{s}$), the VCO is locked on the H-Maser clock simulated but not as well as we want and (at $\mathrm{t} = 3 \cdot 10^5\mathrm{s}$) on the Cs clock simulated.
At the moment of our study, we are trying to perform these simulations. For that, we must adjust all the three parameters : the couple $(\mathrm{b_0}, \mathrm{b_1}), \mathrm{K}$ and s. Note that, the treatement is too long because we use many data $(\simeq 10^{10} data)$ for each clock. But soon new results will be available.

Conclusion

We have presented in this paper the theory and the effect of the differents parameters on our servoing loop. And like in any servoing, we have to make choices according to our needs. The trio (s, K and couple $(b_0,\ b_1)$) take a very important place in this digital servoing because it acts on the stability of the system.